HW3 Written Questions

# 4.1 Problem 1

Read Section 9.1 in Bishop that discusses the K-means algorithm, and solve problem 9.1 which asks you to prove that it converges.

Consider the K-means algorithm discussed in Section 9.1. Show that as a consequence of there being a finite number of possible assignments for the set of discrete indicator variables rnk and that for each such assignment there is a unique optimum for the {μk} the K-means algorithm must converge after a finite number of iterations.

**Answer**:

If there are N data points which are assigned to K clusters. There will be possible solutions, which is a finite number. Each iteration, we attempt to lower the J function (9.1) to update parameters { rnk } and { μk } for new clustering based on the previous clustering. At last, we finally will get a clustering which will keep the same parameters in the following iterations, which means the algorithm finally converge in a finite number of iterations.

# 4.2 Problem 2

Read the beginning of Section 9.2 which describes Gaussian mixture models, and solve Problem 9.3.

Consider a Gaussian mixture model in which the marginal distribution p(z) for the latent variable is given by (9.10) and the conditional distribution p(x|z) for the observed variable is given by (9.11). Show that the marginal distribution p(x) obtained by summing p(z)p(x|z) over all possible values of z is a Gaussian mixture of the form (9.7).

**Answer**:

Since z is binary 1-of-K coding variable where , which mean from 1 to K, there is only one k makes zk=1, otherwise, z=0.

Thus, marginal distribution of z (9.10) can also be written as:

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Where only when zk=1, the exponent of will be one, otherwise the exponent will be 0.

Then, when comes p(x)

# 4.3 Problem 3

Go through Section 12.1.2 which describes the Minimum-error formulation of PCA and perform omitted computations. Specifically, do all the derivations necessary to show that

0. Before (12.9) αnj =xTnuj

1. (12.12) znj =xTnuj

2. (12.13) bj = xTuj

3. In case of two-dimensional data space

Su2 = λ2u2

J = λ2

Answer:



